Conformal p-branes as a Source of Structure in Spacetime

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Abstract

We discuss a model of a conformal p-brane interacting with the world volume metric and connection. The purpose of the model is to suggest a mechanism by which gravity coupled to p-branes leads to the formation of *structure* rather than homogeneity in spacetime. Furthermore, we show that the formation of structure is accompanied by the appearance of a *multivalued* cosmological constant, i.e., one which may take on different values in different domains, or *cells*, of spacetime. The above results apply to a broad class of non linear gravitational lagrangians as long as metric and connection on the p-brane manifold are treated as independent variables.

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I. INTRODUCTION

This paper has two main objectives. The first is to discuss a new effect that the dynamics of extended objects (p–branes) may have on the geometry of spacetime. The second, allied, objective is to introduce a class of gravity theories in (p+1)–dimensions characterized by the formation of structure on the p–brane manifold.

To elaborate further on these points, and partly to motivate our work, we recall one of the basic tenets of General Relativity, namely, that the matter content of the universe shapes the spacetime geometry and, conversely, that the geometry " guides " the motion of material *particles* along geodesic lines. The notion of " geodesic motion " can be generalized to incorporate the world–track swept by *extended objects*, say strings and membranes, in curved spacetime [1]. However, once this is done, one finds that a new possibility arises from the interplay between geometry and dynamics, namely, the *formation of structure in spacetime*. This structure consists of separate vacuum domains, or *cells* of spacetime, each characterized by a distinct geometric phase; that is, the background geometry could be Riemannian or Minkowskian in one domain, of Weyl type in another or Riemann–Cartan in yet another cell, etc., with the highest concentration of matter to be found on the domain walls separating each cell. The interior of each cell constitutes a " false vacuum " to the extent that it is characterized by a distinct value of the cosmological constant or vacuum energy density. Since each cell has a dynamics of its own, this overall cellular structure is an ever changing one, and the qualitative picture that comes to our mind is that of a " frothiness " in the very fabric of spacetime.

In different guises and with different objectives in mind, a cellular structure in spacetime has been invoked before [2], and is implicitly assumed, for instance, in connection with the idea of chaotic inflation [3], or in connection with the geometrodynamic idea of *spacetime foam* as the inevitable consequence of quantum fluctuations in the gravitational field [4]. However, since very little is known about quantum gravity, a mathematical implementation of these ideas has always been exceedingly difficult, or vague. *In contrast, the aim of this paper is to show that one can " tunnel through the barrier of ignorance " about quantum gravitational effects and discuss the formation of a multiphase cellular structure in spacetime as a consequence of the classical dynamics of p–branes coupled to gravity.* This brings us to our second and more detailed objective, i.e., the introduction of a new class of gravity theories in (p+1)–dimensions. To trace the genesis of this approach we recall that the notion of a spacetime with many geometric phases originated in an earlier attempt to deal with the phenomena of vacuum decay and inflation [5], and, more recently, from a stochastic approach to the dynamics of a string network in which we have shown that domains of spacetime (voids) characterized by a Riemannian geometry and a nearly uniform string distribution, appear to be separated by domain walls characterized by a Weyl type geometry and by a discontinuity in the string distribution [6]. In this approach, based on a stochastic interpretation of the Nambu–Goto action, the geometry of spacetime is not preassigned, but it is required to be compatible with the matter–string distribution in the universe. Thus, the string degrees of freedom are coupled to both metric γ_{mn} and connection $\Gamma^l{}_{mn}$ of the *ambient spacetime* through the curvature scalar $R = \gamma^{mn} R_{mn}(\Gamma)$. The general philosophy of this paper is the same, i.e., geometry and matter distribution must be self consistent and not preordained. In this paper, however, by " geometry of spacetime " we mean the *intrinsic*

geometry of the Lorentzian p–brane manifold and not the geometry of the target space in which the p–brane is imbedded and which we assume, for simplicity, to be a D–dimensional Minkowski spacetime. From this vantage point, the p–brane *classical* action that we suggest below in Eqs. (2.1 and 2.5), can be interpreted as the action for gravity in (p+1)–dimensions coupled to some " scalar fields " represented in the action by the imbedding functions with support on the p–brane manifold. The payoff of this particular choice of action is the possibility, not usually contemplated by conventional General Relativity, of a multiphase intrinsic geometry that may form on a p–brane manifold. In fact, the main result of this paper is a mechanism, *coded in the condition* (2.14), by which gravity coupled to *extended objects* manages to produce structure rather than uniformity in spacetime. The source of this mechanism can be traced back to two key properties of our model: the first property is that the gravitational term in the action is described by an it analytic function of the scalar curvature on the p–brane manifold; the second property is that the energy–momentum tensor of the p–brane is *traceless.*These properties are coded in the two terms of the action (2.5): the first property (analyticity of the gravitational term) is simply *assumed*, with no other justification tha to serve our purpose, which is to arrive at the condition (2.14) bypassing quantum gravitational effects; the second property (tracelessness of the energy–momentum tensor) is *enforced* by restricting our consideration to *conformal p– branes* defined by Eq.(2.1). The rationale for this choice of p– brane action is that any other choice would result in the appearance of the trace of the energy–momentum tensor on the right hand side of Eq.(2.14), thereby invalidating our conclusions.

The main body of the paper, section II, is divided into three subsections. In subsection A, we introduce the action functional for the conformal p–brane non–minimally coupled to the world volume metric and connection, which we consider as independent variables. In subsection B, we describe the solution of the classical field equations corresponding to a Riemannian geometry over the p–brane world volume. In subsection C, we show how the same classical field equations admit another type of solution. For a generic p–brane, with $p > 1$, this solution corresponds to a Riemann–Cartan geometry characterized by a traceless torsion tensor. The string case is exceptional in that the solution of the field equations corresponds to a Weyl geometry.

II. THE ACTION

A. Classical p–brane dynamics

In the conventional approach originated by Dirac, Nambu and Goto, p–branes are treated as $(p + 1)$–dimensional manifolds *imbedded* in a D–dimensional spacetime. Alternatively, one may elect to focus on the *intrinsic geometry* of the p–brane manifold, regardless of the imbedding in the ambient spacetime. Our action integral reflects both points of view. A first step toward this " hybrid " model was suggested for the string by Howe and Tucker [7]. On purely dimensional grounds, the Howe–Tucker string action, which is equivalent to that of Nambu and Goto, is invariant under Weyl rescaling of the world metric γ_{mn} and, as a consequence, the string classical energy–momentum tensor has vanishing trace. *This is the key property of strings which we wish to extend to a generic p–brane.*As anticipated

in the Introduction, one way to achieve this is to give up the world–volume interpretation of the action and to formulate p–brane dynamics in a manifestly Weyl invariant form. The extension of the Howe–Tucker action, though feasible, does *not* meet this requirement [8]. Rather, the Weyl invariant classical action for a p–brane is [9]

$$S_C = -\kappa \int_W d^{p+1}\xi \sqrt{-\gamma} \left[\frac{1}{(p+1)} \gamma^{mn} \partial_m X^\mu \partial_n X_\mu \right]^{(p+1)/2} , \tag{2.1}$$

where κ is the p–brane surface tension, ξ^m, $m = 0, \ldots, p$ denote the world volume coordinates with world volume metric γ_{mn}, and X^μ, $\mu = 0, \ldots, D-1$, denote spacetime coordinates with a flat metric $\eta_{\mu\nu}$. Since the combination $\sqrt{-\gamma}\,(\gamma^{mn})^{(p+1)/2}$ is Weyl invariant for any p, the p–brane energy momentum tensor

$$\begin{aligned}
T_{mn} &\equiv -\frac{2}{\sqrt{-\gamma}} \frac{\delta S_C}{\delta \gamma^{mn}} \\
&= \kappa \partial_m X^\rho \partial_n X_\rho \left[\frac{1}{(p+1)} \gamma^{pq} \partial_p X^\mu \partial_q X_\mu \right]^{(p-1)/2} + \\
&- \kappa \gamma_{mn} \left[\frac{1}{(p+1)} \gamma^{pq} \partial_p X^\mu \partial_q X_\mu \right]^{(p+1)/2}
\end{aligned} \tag{2.2}$$

is *traceless*, i.e. $T^m{}_m = 0$.

The next step in our approach is to add to the action an explicit symmetry breaking term which accounts for the " intrinsic " gravitational interaction on the p–brane manifold. Note that a generic term of this type is expected to arise in the effective action as a consequence of quantum corrections [10]. However, for our specific purposes, stated in the Introduction, we define on the p– brane manifold a world hypersurface *affine connection* $\Gamma^s{}_{mn}$, $(s, m, n = 0, \ldots, p)$ through an interaction term $L_{int.}(R)$ which is assumed to be an *analytic*, but otherwise arbitrary function of the world volume scalar curvature R. We do not select the usual Christoffel connection because this choice would impose a Riemannian geometry on the world volume. Instead, as explained in the Introduction, we consider the conformal p–brane geometry as a dynamical quantity to be determined by the equations of motion. In this general case, the strength of the connection is measured by the curvature tensor

$$R^l{}_{mns} \equiv \partial_n \Gamma^l{}_{ms} - \partial_s \Gamma^l{}_{mn} + \Gamma^l{}_{an} \Gamma^a{}_{ms} - \Gamma^l{}_{as} \Gamma^a{}_{mn} , \tag{2.3}$$

and the corresponding contracted curvature tensor and curvature scalar, are given by

$$R_{ms}(\Gamma) = R^l{}_{mls} , \qquad R(\gamma, \Gamma) = \gamma^{ms} R_{ms} . \tag{2.4}$$

Note that R_{ms} does *not* depend on the world volume metric, but is a function of the connection alone. Futhermore, γ^{mn} projects out the symmetric part of R_{mn} in the definition of the scalar curvature R. Against this background, the action describing our model is

$$S(X, \gamma, \Gamma) = S_C(X, \gamma) + \int_W d^{p+1}\xi \sqrt{-\gamma} L_{int.}(R) \tag{2.5}$$

in which $L_{int.}(R)$ can be regarded either as an assigned function of R, or as a generic analytic function to be determined by the equations of motion. Note that in the action (2.1), the

p–brane is *minimally* coupled to the world volume metric in a Weyl invariant manner, whereas in the action (2.5) we have introduced a non–minimal interaction term. Except for a special form of $L_{int.}(R)$, to be discussed shortly, it is to be expected that this term breaks the conformal symmetry of the action (2.1) and our immediate objective is to discuss the main dynamical consequence of this symmetry breaking term, namely, the formation of structure accompanied by the appearance of a multivalued cosmological constant on the p–brane manifold.

Varying eq. (2.5) with respect to the p–brane coordinates X^μ , we find

$$\partial_m \left(\sqrt{-\gamma} \gamma^{mn} \partial_n X^\mu \right) = 0 \ . \tag{2.6}$$

Equation (2.6) is the " free " wave equation for the p–brane field $X^\mu(\xi)$ and would represent the whole content of our model in the absence of the intrinsic gravitational term. Eq (2.6) is essentially a generally covariant Klein–Gordon equation with respect to the world volume metric γ_{mn}, and does not depend on the connection Γ. As a matter of fact, $X^\mu(\xi)$ behaves as a scalar multiplet under a general coordinate transformation $\xi^m \to \xi'^m = \xi'^m(\xi)$, and, therefore, general covariance only determines the coupling to the metric.

Next, varying eq. (2.5) with respect to the world volume metric, we find

$$\gamma^{mn} \left[\frac{1}{(p+1)} \gamma^{pq} \partial_p X^\mu \partial_q X_\mu \right]^{(p+1)/2} - \gamma^{mi} \gamma^{nj} \partial_i X^\rho \partial_j X_\rho \left[\frac{1}{(p+1)} \gamma^{pq} \partial_p X^\mu \partial_q X_\mu \right]^{(p-1)/2} +$$

$$-L'_{int.}(R) R^{(mn)}(\Gamma) + \frac{1}{2} L_{int.}(R) \gamma^{mn} = 0 \ , \tag{2.7}$$

where the prime denotes derivation with respect to R, $R_{(mn)}$ is the symmetric part of the contracted curvature tensor, and ∇_a is the covariant derivative with respect to the Γ connection. In the absence of non–minimal interactions, eq. (2.7) reduces to a relationship between the world volume metric γ_{mn} and the induced metric $g_{mn} = \partial_m X^\mu \partial_n X_\mu$, modulo an arbitrary Weyl rescaling. This relationship is changed by $L_{int.}(R)$ and eq.(2.7) encodes the coupling between the p–brane field, metric and connection in the general case.

Finally, we have to vary the action with respect to the connection. In order to do this, it may be useful to recall the formula

$$\gamma^{ms} \delta_\Gamma R_{ms}(\Gamma) = \gamma^{ms} \left[\nabla_l \delta \Gamma^l{}_{ms} - \nabla_s \delta \Gamma^l{}_{ml} \right] \ . \tag{2.8}$$

Hence, the requirement

$$\delta_\Gamma S = \int_W d^{p+1} \xi \sqrt{-\gamma} L'_{int.}(R) \gamma^{ms} \delta_\Gamma R_{ms}(\Gamma) = 0 \tag{2.9}$$

gives, after an integration by parts:

$$\nabla_l \left[\sqrt{-\gamma} L'_{int.}(R) \gamma^{mn} \right] - \nabla_s \left[\sqrt{-\gamma} L'_{int.}(R) \gamma^{ms} \right] \delta^n_l = 0 \ . \tag{2.10}$$

Taking the trace over the pair (l, n), we find that $\nabla_n(\sqrt{-\gamma} L'_{int.}(R) \gamma^{mn}) = 0$, so that we can write eq. (2.10) in the form

$$\nabla_l \left[L'_{int.}(R) \sqrt{-\gamma} \gamma^{mn} \right] = 0 \ . \tag{2.11}$$

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Equation (2.11) relates $\Gamma^m{}_{nr}$ to γ_{mn} and can be used to determine the world volume geometry. In order to see this, we note that the first two terms in eq.(2.7) represent just the traceless p–brane energy– momentum tensor (2.2). Therefore, if we take the trace of eq. (2.7), the dependence on $X^\mu(\xi)$ disappears and we obtain the following relation between the metric and the connection,

$$RL'_{int.}(R) - \frac{p+1}{2}L_{int.}(R) = 0. \tag{2.12}$$

Equation (2.12) was first derived in ref. [11] as a condition on a broad class of non–linear gravitational lagrangians leading to the same Einstein equations obtained from the usual Hilbert action. Volovich [12] has subsequently applied that condition to the case of gravity on the world–sheet of a string and our work was largely inspired by these papers. Regarding equation (2.12), essentially one has two options: the first is to interpret eq.(2.12) as a *differential* equation for $L_{int.}$, in which case the solution is easily found to be

$$L_{int.}(R) = \text{const.} \times R^{(p+1)/2} \ . \tag{2.13}$$

This function is analytic and invariant under Weyl rescaling. Thus, for any extended object, there is a non–minimal gravitational coupling which is singled out by the Weyl invariance of the action. However, in general one starts from an *assigned*, non–invariant interaction Lagrangian, so that the form of eq. (2.12) is fixed *a priori*. This is our second option. As an example, if we specialize the model to the *bag* case, $p = 3$, a suggestive form of $L_{int.}(R)$ is: $L_{int.}(R) = \rho - \mu^2 R(\Gamma) + \lambda R^2(\Gamma)$. This " interaction " lagrangian can be interpreted as first order General Relativity plus a quadratic correction in which ρ plays the role of the " bare " cosmological constant and μ can be identified with the Planck mass. Note that if we set $R(\Gamma) \equiv \phi^2$, where ϕ is a scalar field, then $L_{int.}(R)$ takes the form of a *Higgs potential*, and one may wonder about spontaneous symmetry breaking of Weyl invariance. However, in spite of this formal similarity, one should keep in mind that Weyl invariance is broken explicitly, rather than spontaneously, by the very presence of an interaction term, regardless of the specific form of $L_{int.}(R)$.

Returning to the general case and to the formation of structure, we suggest to interpret $L'_{int.}$ in equation (2.12) as an *order parameter* for the geometric phases on the p–brane manifold. The essential property which makes this interpretation possible is that an analytic function has a *discrete* number of zeros within its analyticity domain. Since the whole left hand side of eq. (2.12) is an analytic function of R, there can be only a discrete set of solutions, say $\{c_i\}$, such that

$$c_i L'_{int.}(c_i) - \frac{p+1}{2}L_{int.}(c_i) = 0 \ , \qquad R = c_i \ . \tag{2.14}$$

Hence, the conformal p–brane geometry admits two distinct phases characterized by the " order parameter " $L'_{int.}(c_i) = 0$, or, $L'_{int.} \neq 0$. In the first instance , the scalar curvature $R = c_i$ is an extremal of L_{int} and eq.(2.14) implies $L_{int}(c_i) = 0$. When $L'_{int}(c_i) \neq 0$, eq.(2.14) implies $L_{int}(c_i) \neq 0$. We will argue, next, that in correspondence of each of these cases there exists a distinct geometric phase with a characteristic cellular structure on the p–brane manifold.

B. Riemannian geometric phase

If the curvature extremizes the " potential ", i.e. $L'_{int.}(c_i) = 0$, then equation (2.14) requires $L_{int.}(c_i) = 0$. Equation (2.11) is trivially satisfied, and the connection is no longer dynamically determined but can be freely chosen. In this case, eq.(2.7) simplifies and becomes,

$$\gamma^{mn} \left[\frac{1}{(p+1)} \gamma^{pq} \partial_p X^\mu \partial_q X_\mu \right]^{(p+1)/2} +$$

$$-\gamma^{mi}\gamma^{nj}\partial_i X^\rho \partial_j X_\rho \left[\frac{1}{(p+1)} \gamma^{pq} \partial_p X^\mu \partial_q X_\mu \right]^{(p-1)/2} = 0. \tag{2.15}$$

From this equation it follows that the world volume metric can be written as the induced metric times an arbitrary function of the world coordinates,

$$\gamma_{mn} = \Omega(\xi)\partial_m X^\mu \partial_n X_\mu . \tag{2.16}$$

Thus, this geometric phase corresponds to a Riemannian background geometry which is governed by the first order, contracted, Einstein equation $R = c_i$. Evidently, *for each* c_i, this equation describes a spacetime of constant curvature (p–cell). Thus, barring any degeneracy in the set of solutions $\{c_i\}$, one is led to the conclusion that the dynamics of a p–brane induces a cellular structure on the p–brane manifold. For $p = 3$, each cell consists of a three dimensional region separated from other cells by domain walls and the over all structure resembles an " emulsion " [5], or a " soap bubble froth " in which the dynamics of each bubble is governed by matching conditions on the metrics of neighboring cells.

Note, incidentally, that the contracted Einstein equation $R = c_i$ represents a generalization of the basic equation of (1+1)– dimensional gravity. As a matter of fact Eq.(2.16) holds true for any p–brane and is a consequence of the Weyl invariance of the p–brane action. However, while conformal invariance allows a common formal treatment of strings and higher dimensional objects, the *role* played by conformal invariance is distinctly unique in the case of strings. For instance, equation (2.16) does *not* imply that the p–brane manifold is conformally flat except in the string case, $p = 1$, for which one can find a coordinate transformation which maps the induced metric into a flat metric. A necessary and sufficient condition for *conformal flatness* of higher dimensional manifolds with $p + 1 \geq 4$ is that the Weyl tensor vanishes.

C. Riemann–Cartan geometric phase

If $L'_{int.}(c_i) \neq 0$ then $\Gamma^m{}_{nr}$ becomes a dynamical variable. In fact, eq. (2.11) gives

$$\nabla_a \left[\sqrt{-\gamma}\gamma^{mn} \right] = 0 \longrightarrow \nabla_a \gamma^{mn} = \frac{\gamma^{mn}}{\sqrt{-\gamma}} \nabla_a \sqrt{-\gamma}. \tag{2.17}$$

But,

$$\nabla_a \sqrt{-\gamma} = \frac{1}{2}\sqrt{-\gamma}\left[\gamma^{mn}\partial_a\gamma_{mn} - 2\Gamma^m{}_{ma}\right] \; , \tag{2.18}$$

so that eq. (2.17) can be written in the form

$$\nabla_a\gamma^{mn} = \frac{1}{2}\gamma^{mn}\left[\gamma^{rs}\partial_a\gamma_{rs} - 2\Gamma^l{}_{la}\right] \; . \tag{2.19}$$

To solve eq. (2.19), we recall that a general affine connection can always be written as the Christoffel symbol plus a term, say $K^l{}_{mn}$, which behaves as a tensor under general coordinate transformation

$$\Gamma^l{}_{mn} = \{_m{}^l{}_n\} + K^l{}_{mn} \; . \tag{2.20}$$

The Christoffel symbol $\{_m{}^l{}_n\}$ is a metric compatible connection, so that the ansatz (2.20), once inserted into eq. (2.19), gives us an equation for the tensor part $K^l{}_{mn}$ alone

$$(p-1)K^l{}_{lm} = 0 \; , \tag{2.21}$$

where we have used the identity $\{_m{}^l{}_l\} = (1/2)\gamma^{ab}\partial_m\gamma_{ab}$. Eq. (2.21) shows that, for any extended object different from the string, the trace of $K^l{}_{mn}$ must vanish, so that eq. (2.19) for the ansatz (2.20) reduces to

$$\nabla_a\gamma^{mn} = 0 \implies K^l{}_{pq} = \frac{1}{2}\left(T^l{}_{pq} + T_{pq}{}^l + T_{qp}{}^l\right) \; , \tag{2.22}$$

where $T^l{}_{pq} = (1/2)\left(\Gamma^l{}_{pq} - \Gamma^l{}_{qp}\right)$ is the *torsion tensor*, and $\Gamma^l{}_{pq}$ is identified with the *Riemann–Cartan connection*. This new geometric phase is also characterized by a cellular structure, since the scalar curvature is still subject to the constraint $R(\gamma,\Gamma) = c_i$. The novelty in this case is the appearance of a cosmological constant with a cell–dependent value. Indeed, in this geometric phase, eq. (2.7) reduces to the Einstein–Cartan field equation

$$R^{(mn)}(\Gamma) - \frac{c_i}{p+1}\gamma^{mn} = -\frac{1}{L'_{int.}(c_i)}T^{mn}(X) \; , \tag{2.23}$$

where $-(1/L'_{int.}(c_i))$ plays the role of Newton's constant, and $c_i/(p+1)$ acts as an effective cosmological constant in any given cell on the p–brane manifold. It is interesting how Newton's constant and the cosmological constant are related by the above formalism. Evidently both originate from the set of solutions $\{c_i\}$ of equation (2.14) (analyticity assumption). As anticipated in the Introduction, it is this assumption that allows us to bypass our ignorance of quantum gravitational effects: if a generic p–cell has a linear dimension of the order of Planck's length at the time of its nucleation, then the analyticity assumption is tantamount to state that the quantum fluctuations in the background metric are of the same order of magnitude as the metric itself, which is the central consideration behind the geometrodynamic idea of spacetime foam. Once the nucleation of p–cells has taken place, the problem of their evolution is largely a classical and tractable one [13], and this is the point of view advocated in this paper.

Finally, it should be noted that our formalism also provides an insight into the question of the special status that strings hold among p–branes: the point is that, for $p = 1$, eq.(2.21)

is satisfied by *any* $K^l{}_{lm}$. This means that the connection $\Gamma^l{}_{qp}$ is defined up to an arbitrary vector field $B_m \equiv -K^l{}_{lm}$. Accordingly, eq.(2.19) becomes

$$\nabla_a \gamma^{mn} = \gamma^{mn} B_a \ . \tag{2.24}$$

Eq.(2.24) is the *semi–metric* condition for the Riemann–Weyl connection [12]

$$\Gamma^l{}_{qp} = \{{}^{\ l}_{q\ p}\} + \frac{1}{2}\left(\delta^l_p B_q + \delta^l_q B_p - \gamma_{pq} B^l\right) \tag{2.25}$$

where B_p acts as the Weyl " gauge potential " associated with volume–changing scalings.

Thus, we conclude that the intrinsic geometry on the world–sheet of a string is characterized by the pair (γ_{mn}, B_p), while for a generic p–brane the geometrical objects are the metric and a traceless torsion tensor. Furthermore, the above results seem to be independent of any special length or energy scale but seem suggestive enough to be given a cosmological interpretation *at, or near the Planck scale* in the physically interesting case in which the p–brane consists of a spatial 3–dimensional manifold, $p = 3$. In this case, the non– minimal coupling term to the bag curvature gives rise to a " gravitational action " whose effect is to form a cellular structure on the manifold. This structure is not static, but a highly dynamical one which evokes, at least in our mind, a vivid picture of the ground state of the primordial universe not unlike the chaotic inflation scenario [3]. In the light of the above results, the physical spacetime can be pictured as a set of cells in which the geometry is dynamically determined and not fixed at the outset. In this scenario, extended objects (strings and membranes) may well play a role comparable, or even alternative, to that of the Higgs field, as the universe bootstraps itself into existence out of the primordial spacetime foam. In this paper we have suggested that this structure is a manifestation of the underlying multiphase geometry induced by the very dynamics of p–branes encoded in the action (2.5). In this interpretation, the cosmic vacuum is a multi–phase system in a double sense: inside a cell there may exist a Riemannian or a Riemann–Cartan geometry; furthermore, for each type of geometry, curvature can attain different constant values labelled by c_i. These parameters, in turn, determine the value and sign of the energy density in each cell. Consequently, each cell may behave as a blackhole, wormhole, inflationary bubble, etc.. The classical and semiclassical evolution of any such cell has been discussed in earlier papers [13]. Here, as a final note, we add that a semi–classical description of the quantum mechanical ground state, for such a multi–domain system, is obtained by approximating the (euclidean) Feynman integral with the sum over classical solutions. The non–minimal interaction term in eq. (2.5) acts as an *effective cosmological constant* once evaluated on a classical solution. Thus, the cosmological constant enters the model as a semi–classical dynamical variable and, therefore, is susceptible of dynamical adjustments [10]. From this view point, the vanishing of $L_{int.}(c_i)$ in the Riemannian phase is an attractive result.

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